3-1 Solving Systems of Equations by Graphing

Main Ideas

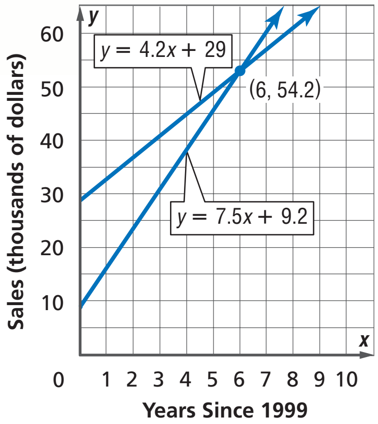
- Solve systems of linear equations by graphing.
- Determine whether a system of linear equations is consistent and independent, consistent and dependent, or inconsistent.

New Vocabulary

system of equations
consistent
inconsistent
independent
dependent

GET READY for the Lesson

Since 1999, the growth of in-store sales for Custom Creations can be modeled by $y = 4.2x + 29$. The growth of their online sales can be modeled by $y = 7.5x + 9.2$. In these equations, x represents the number of years since 1999, and y represents the amount of sales in thousands of dollars.



The equations $y = 4.2x + 29$ and $y = 7.5x + 9.2$ are called a system of equations.

Solve Systems Using Tables and Graphs A **system of equations** is two or more equations with the same variables. To solve a system of equations, find the ordered pair that satisfies all of the equations.

EXAMPLE 1 Solve the System of Equations by Completing a Table

Solve the system of equations by completing a table.

$$-2x + 2y = 4$$
$$-4x + y = -1$$

Write each equation in slope-intercept form.

$$-2x + 2y = 4 \rightarrow y = x + 2$$
$$-4x + y = -1 \rightarrow y = 4x - 1$$

Use a table to find the solution that satisfies both equations.

x	$y_1 = x + 2$	y_1	$y_2 = 4x - 1$	y_2	(x, y_1)	(x, y_2)
−1	$y_1 = (-1) + 2$	1	$y_2 = 4(-1) - 1$	−5	(−1, 1)	(−1, −5)
0	$y_1 = 0 + 2$	2	$y_2 = 4(0) - 1$	−1	(0, 2)	(0, −1)
1	$y_1 = (1) + 2$	3	$y_2 = 4(1) - 1$	3	(1, 3)	(1, 3)

The solution of the system of equations is the ordered pair that satisfies both equations.

The solution of the system is (1, 3).

CHECK Your Progress

1A. $-3x + y = 4$
$2x + y = -6$

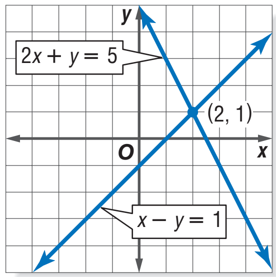
1B. $2x + 3y = 4$
$5x + 6y = 5$

Another way to solve a system of equations is to graph the equations on the same coordinate plane. The point of intersection represents the solution.

EXAMPLE Solve by Graphing

2 **Solve the system of equations by graphing.**

$$2x + y = 5$$
$$x - y = 1$$



Write each equation in slope-intercept form.

$2x + y = 5 \rightarrow y = -2x + 5$

$x - y = 1 \rightarrow y = x - 1$

The graphs appear to intersect at (2, 1).

CHECK Substitute the coordinates into each equation.

$2x + y = 5$	$x - y = 1$	Original equations
$2(2) + 1 \stackrel{?}{=} 5$	$2 - 1 \stackrel{?}{=} 1$	Replace *x* with 2 and *y* with 1.
$5 = 5$ ✓	$1 = 1$ ✓	Simplify.

The solution of the system is (2, 1).

Study Tip

Checking Solutions

When using a graph to find a solution, always check the ordered pair in *both* original equations.

CHECK Your Progress

2A. $4x + \frac{1}{3}y = 8$
$3x + y = 6$

2B. $5x + 4y = 7$
$-x - 4y = -3$

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Systems of equations are used in businesses to determine the *break-even point*. The break-even point is the point at which the income equals the cost.

Real-World EXAMPLE Break-Even Point Analysis

Real-World Link

Compact discs (CDs) store music digitally. The recorded sound is converted to a series of 1s and 0s. This coded pattern can then be read by an infrared laser in a CD player.

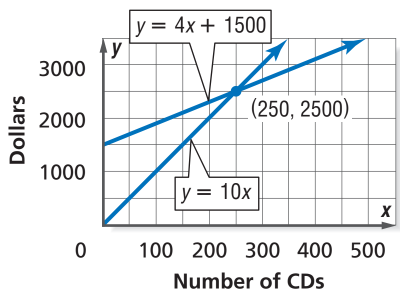
3 **MUSIC The initial cost for Travis and his band to record their first CD was $1500. Each CD will cost $4 to produce. If they sell their CDs for $10 each, how many must they sell before they make a profit?**

Let x = the number of CDs and let y = the number of dollars.

Costs of CDs	is	cost per CD	plus	start-up cost.
y	$=$	$4x$	$+$	1500

Income for CDs	is	price per CD	times	number sold.
y	$=$	10	$\cdot$	x

The graphs intersect at (250, 2500). This is the break-even point. If the band sells fewer than 250 CDs, they will lose money. If the band sells more than 250 CDs, they will make a profit.



CHECK Your Progress

3A. **RUNNING** Curtis will run 4 miles the first week of training and increase the mileage by one mile each week. With another schedule, Curtis will run 1 mile the first week and increase his total mileage by 2 miles each week. During what week do the two schedules break even? How many miles will Curtis run during this week?

Study Tip

Graphs of Linear Systems

Graphs of systems of linear equations may be intersecting lines, parallel lines, or the same line.

Classify Systems of Equations A system of equations is **consistent** if it has at least one solution and **inconsistent** if it has no solutions. A consistent system is **independent** if it has exactly one solution or **dependent** if it has an infinite number of solutions.

EXAMPLE Intersecting Lines

4 **Graph the system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.**

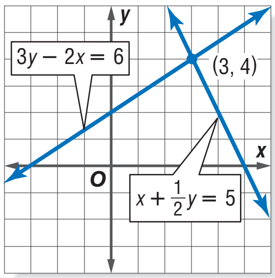
$$x + \frac{1}{2}y = 5$$

$$3y - 2x = 6$$

Write each equation in slope-intercept form.

$$x + \frac{1}{2}y = 5 \quad \rightarrow \quad y = -2x + 10$$

$$3y - 2x = 6 \quad \rightarrow \quad y = \frac{2}{3}x + 2$$



The graphs intersect at (3, 4). Since there is one solution, this system is *consistent and independent*.

CHECK Your Progress

4A. $2x - y = 5$

$x + 3y = 6$

4B. $2x - y = 5$

$y + \frac{1}{2x} = 5$

The graph of a system of linear equations that is consistent and dependent is one line.

EXAMPLE Same Line

5 **Graph the system of equations and describe it as *consistent* and *independent*, *consistent and dependent*, or *inconsistent*.**

$$9x - 6y = 24$$

$$6x - 4y = 16$$

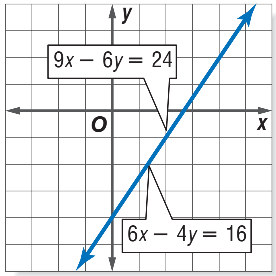
Write each equation in slope-intercept form.

$$9x - 6y = 24 \quad \rightarrow \quad y = \frac{3}{2}x - 4$$

$$6x - 4y = 16 \quad \rightarrow \quad y = \frac{3}{2}x - 4$$

Since the equations are equivalent, their graphs are the same line. Any ordered pair representing a point on that line will satisfy both equations.

So, there are infinitely many solutions to this system. It is *consistent and dependent*.



CHECK Your Progress

5A. $5x - 3y = -2$
$4x + 2y = 5$

5B. $4x + 2y = 5$
$2x + y = \frac{5}{2}$

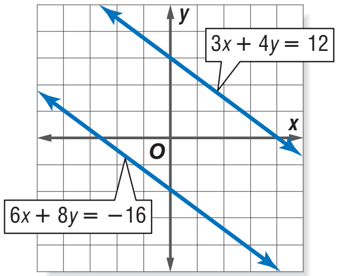
EXAMPLE 6 Parallel Lines

Graph the system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

$\mathbf{3x + 4y = 12}$
$\mathbf{6x + 8y = -16}$

$3x + 4y = 12 \rightarrow y = -\frac{3}{4}x + 3$

$6x + 8y = -16 \rightarrow y = -\frac{3}{4}x - 2$



The lines do not intersect. Their graphs are parallel lines. So, there are no solutions that satisfy both equations. This system is *inconsistent*.

Study Tip

Parallel Lines

Notice from their equations that the lines have the same slope and different *y*-intercepts.

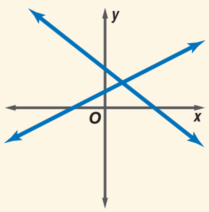
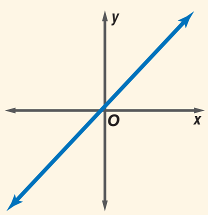
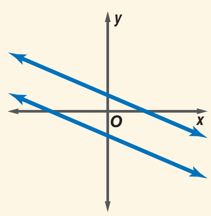
CHECK Your Progress

6A. $y - \frac{4}{3}x = -2$
$y + \frac{3}{4}x = -2$

6B. $y - \frac{4}{3}x = -2$
$y - \frac{4}{3}x = 3$

The relationship between the graph of a system of equations and the number of its solutions is summarized below.

CONCEPT SUMMARY

consistent and independent	consistent and dependent	inconsistent
		
intersecting lines; one solution	same line; infinitely many solutions	parallel lines; no solution

CHECK Your Understanding

Example 1 (p. 116)

Solve each system of equations by completing a table.

1. $y = 2x + 9$
$y = -x + 3$

2. $3x + 2y = 10$
$2x + 3y = 10$

Example 2 (p. 117)

Solve each system of equations by graphing.

3. $4x - 2y = 22$
$6x + 9y = -3$

4. $y = 2x - 4$
$y = -3x + 1$

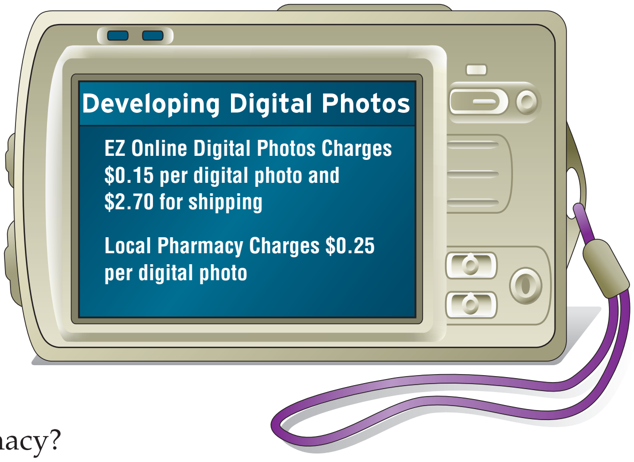
Example 3 (pp. 117–118)

DIGITAL PHOTOS **For Exercises 5–7, use the information in the graphic.**

5. Write equations that represent the cost of printing digital photos at each lab.

6. Under what conditions is the cost to print digital photos the same for either store?

7. When is it best to use EZ Online Digital Photos and when is it best to use the local pharmacy?



Examples 4–6 (pp. 118–119)

Graph each system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

8. $y = 6 - x$
$y = x + 4$

9. $x + 2y = 2$
$2x + 4y = 8$

10. $x - 2y = 8$
$\frac{1}{2}x - y = 4$

Exercises

HOMEWORK HELP

For Exercises	See Examples
11, 12	1
13–18	2
19–26	4–6
27–32	3

Solve each system of linear equations by completing a table.

11. $y = 3x - 8$
$y = x - 8$

12. $x + 2y = 6$
$2x + y = 9$

Solve each system of linear equations by graphing.

13. $2x + 3y = 12$
$2x - y = 4$

14. $3x - 7y = -6$
$x + 2y = 11$

15. $5x - 11 = 4y$
$7x - 1 = 8y$

16. $2x + 3y = 7$
$2x - 3y = 7$

17. $8x - 3y = -3$
$4x - 2y = -4$

18. $\frac{1}{4}x + 2y = 5$
$2x - y = 6$

Graph each system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

19. $y = x + 4$
$y = x - 4$

20. $y = x + 3$
$y = 2x + 6$

21. $x + y = 4$
$-4x + y = 9$

22. $3x + y = 3$
$6x + 2y = 6$

23. $y - x = 5$
$2y - 2x = 8$

24. $4x - 2y = 6$
$6x - 3y = 9$

25. **GEOMETRY** The sides of an angle are parts of two lines whose equations are $2y + 3x = -7$ and $3y - 2x = 9$. The angle's vertex is the point where the two sides meet. Find the coordinates of the vertex of the angle.

26. **GEOMETRY** The graphs of $y - 2x = 1$, $4x + y = 7$, and $2y - x = -4$ contain the sides of a triangle. Find the coordinates of the vertices of the triangle.

EXTRA PRACTICE
See pages 895, 928.
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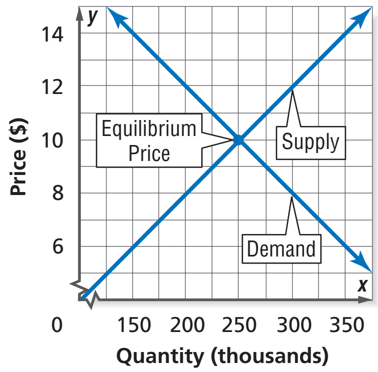
ECONOMICS **For Exercises 27–29, use the graph that shows the supply and demand curves for a new multivitamin.**



In economics, the point at which the supply equals the demand is the *equilibrium price*. If the supply of a product is greater than the demand, there is a surplus and prices fall. If the supply is less than the demand, there is a shortage and prices rise.

27. If the price for vitamins is \$8.00 a bottle, what is the supply of the product and what is the demand? Will prices tend to rise or fall?

28. If the price for vitamins is \$12.00 a bottle, what is the supply of the product and what is the demand? Will prices tend to rise or fall?

29. At what quantity will the prices stabilize? What is the equilibrium price for this product?

Real-World Link

In the United States there is approximately one birth every 8 seconds and one death every 14 seconds.

Source: U.S. Census Bureau

ANALYZE TABLES **For Exercises 30–32, use the table showing state populations.**

Rank	State	Population 2003	Average Annual Gain (2000–2003)
1	California	25,484,000	567,000
2	Texas	22,118,000	447,000
3	New York	19,190,000	70,000
4	Florida	17,019,000	304,000
5	Illinois	12,653,000	80,000

Source: U.S. Census Bureau

30. Write equations that represent populations of Florida and New York x years after 2003. Assume that both states continue to gain the same number of residents every year. Let y equal the population.

31. Graph both equations for the years 2003 to 2020. Estimate when the populations of both states will be equal.

32. Do you think New York will overtake Texas as the second most populous state by 2010? by 2020? Explain your reasoning.

Solve each system of equations by graphing.

33. $\frac{2}{3}x + y = -3$
$y - \frac{1}{3}x = 6$

34. $\frac{1}{2}x - y = 0$
$\frac{1}{4}x + \frac{1}{2}y = -2$

35. $\frac{4}{3}x + \frac{1}{5}y = 3$
$\frac{2}{3}x - \frac{3}{5}y = 5$

Graph each system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

36. $1.6y = 0.4x + 1$
$0.4y = 0.1x + 0.25$

37. $3y - x = -2$
$y - \frac{1}{3}x = 2$

38. $2y - 4x = 3$
$\frac{4}{3}x - y = -2$

Graphing Calculator

To use a TI-83/84 Plus to solve a system of equations, graph the equations. Then, select INTERSECT, which is option 5 under the CALC menu, to find the coordinates of the point of intersection to the nearest hundredth.

39. $y = 0.125x - 3.005$
$y = -2.58$

40. $3.6x - 2y = 4$
$-2.7x + y = 3$

41. $y = 0.18x + 2.7$
$y = -0.42x + 5.1$

H.O.T. Problems

42. **OPEN ENDED** Give an example of a system of equations that is consistent and independent.

43. **REASONING** Explain why a system of linear equations cannot have exactly two solutions.

44. **CHALLENGE** State the conditions for which the system below is: (a) consistent and dependent, (b) consistent and independent, and (c) inconsistent if none of the variables are equal to 0.

$ax + by = c$
$dx + ey = f$

45. ***Writing in Math*** Use the information about sales on page 116 to explain how a system of equations can be used to predict sales. Include an explanation of the meaning of the solution of the system of equations in the application at the beginning of the lesson. How reasonable would it be to use this system of equations to predict the company's online and in-store profits in 100 years? Explain your reasoning.

STANDARDIZED TEST PRACTICE

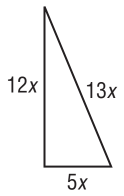
46. **ACT/SAT** Which of the following best describes the graph of the equations?

$4y = 3x + 8$
$-6x = -8y + 24$

- **A** The lines are parallel.
- **B** The lines have the same x-intercept.
- **C** The lines are perpendicular.
- **D** The lines have the same y-intercept.

47. **REVIEW** Which set of dimensions corresponds to a triangle similar to the one shown below?

- **F** 7 units, 11 units, 12 units
- **G** 10 units, 23 units, 24 units
- **H** 20 units, 48 units, 52 units
- **J** 1 unit, 2 units, 3 units

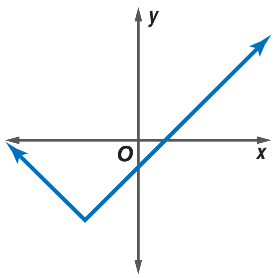


Spiral Review

48. **CHORES** Simon is putting up fence around his yard at a rate no faster than 15 feet per hour. Draw a graph that represents the length of fence that Simon has built. (Lesson 2-7)

Identify each function as S for step, C for constant, A for absolute value, or P for piecewise. (Lesson 2-6)

49.



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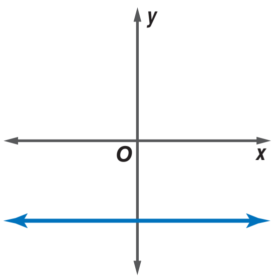
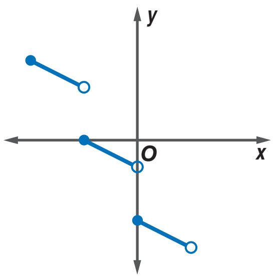


51.



GET READY for the Next Lesson

PREREQUISITE SKILL **Simplify each expression.** (Lesson 1-2)

52. $(3x + 5) - (2x + 3)$

53. $(3y - 11) + (6y + 12)$

54. $(5x - y) + (-8x + 7y)$

55. $6(2x + 3y - 1)$

56. $5(4x + 2y - x + 2)$

57. $3(x + 4y) - 2(x + 4y)$